3. RF Efficiency Calculations. Modern circuit simulators calculate S parameters between defined ports and allow the user to attach programs to the schematic to calculate other parameters based on the calculated S parameters and other desired inputs. The probe circuit will usually have a 50Ω port for each channel. The easiest way to determine η_E is to add high-impedance (monitoring) ports directly across the sample coil, from which the voltage across it can be determined. Often it will also be beneficial to define a number of ports at other points in the circuit to allow voltages at those points to also be monitored.

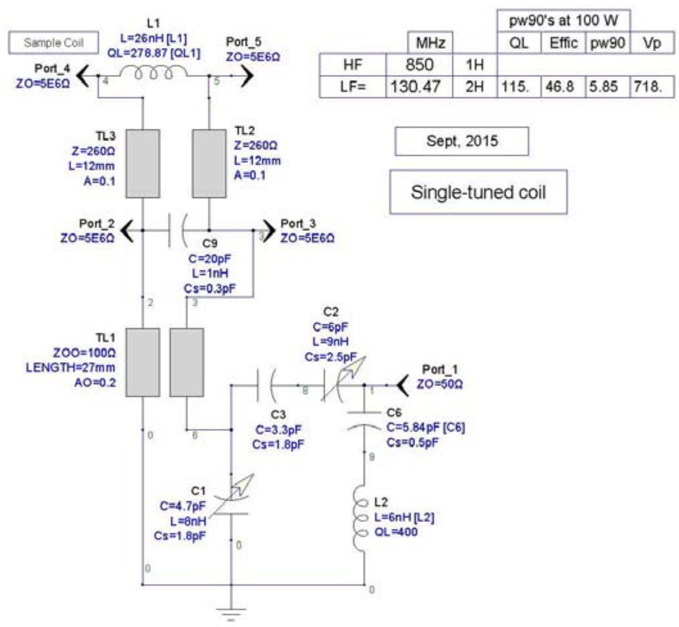

Figure 3.1. A simple single-tuned unbalanced circuit for ^{2}H at 20 T (130.5 MHz).

Figure 3.1 depicts a circuit as used and modeled for an external ^{2}H lock in an 850MHz HXY MAS probe. The 50Ω (LF) rf port is Port_1. The other ports are 5MΩ ports used as voltage probes in the circuit simulation and optimization. Values for rf efficiency (%), matched circuit Q_L, and sample coil peak voltage and pw90 (μs) at 100 W are automatically calculated by the attached program and posted to the upper right corner of the schematic as shown. We use the polarized capacitor symbol for the real capacitor models, which include series L and R and stray capacitance from one side, with some of those values as shown. Transmission line (**TL**) characteristics shown are at 200 MHz and are scaled appropriately with frequency. The coil Q's shown are usually for 150 MHz unless noted otherwise, and scaled appropriately to the pertinent frequency (except for HF traps, where Q's are shown at the HF).

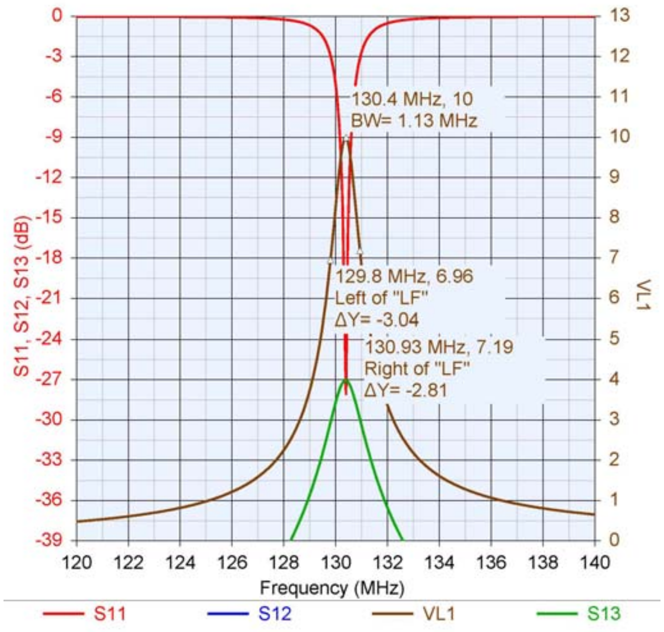

Figure 3.2. Some S parameters (in dB) and VL1 for the circuit of Figure 3.1.

Note that η_E is calculated to be only 46.8% for this simple low-frequency circuit with just 12 mm leads (with attenuation coefficient A=0.1 dB/m at 200 MHz) to the primary tune capacitor C4. (Balancing doesn't help with this small coil.)

The rf peak voltage across the sample coil L1, ports 4 and 5 in Figure 3.1, when the circuit is driven by 1 V peak input (the default when matched) from Port_1 is calculated by the following:

$$V_{L1} = c_v \text{ *mag(S[1,4]-S[1,5])} \qquad (3.1)$$

where c_v is the appropriate normalization factor and the S parameters are in numeric values (not the more common dB relative power). We have chosen to set monitor-port impedances to 5E6 Ω. Thus, for 50Ω input at Port_1, to get volts from the above equation, c_v=(1E5)^0.5=316. (We should point out that while the software includes a voltage gain function, it is not a good method for getting the voltage needed here because of the way phase shifts are handled and the effects of reflections when the port is not perfectly matched. Using S parameters always works.)

However, we have found it more convenient to instead choose c_1 so that the "V" calculated by equation (3.1) is 10.0 at resonance. One reason for our method is that the simulation software we've used doesn't have the ability to search for peaks and report S parameters at the peak to the programs attached by the user. So one must display a graph of "V" and enter its value at the peak, plus the value of c_1. An even more compelling reason is that with multiple traces on the graphs, some referencing a dB scale on the left and some a linear scale on the right, as seen in Figure 3.2, it's much easier if the graph vertical scales stay constant. Keeping a constant peak value of 10 also allows very quick calculation of changes in efficiency when a change is made in the circuit. So we just need to enter the c_1 value needed to display a peak value of 10, and then calculate efficiencies and voltages based on that.

The percent efficiency η_E of the circuit in Figure 3.1 is then calculated by

$$\eta_E = 100 * (c_0 / c_1)^2 \qquad (3.2)$$

where c_0 is the scale factor required to display a peak value of 10.0 for the ideal circuit, in which all components other than the sample coil are lossless, as seen in Figure 3.3. The standard capacitor symbol is used for ideal capacitors. Note that the calculated efficiency is now 100% and the pw90 at 100 W has decreased from 5.85 µs to 4.0 µs. (Also note that Q_L for this circuit is much greater than $Q_{0L}/2$ – because the leads are essentially inductors of infinite Q.)

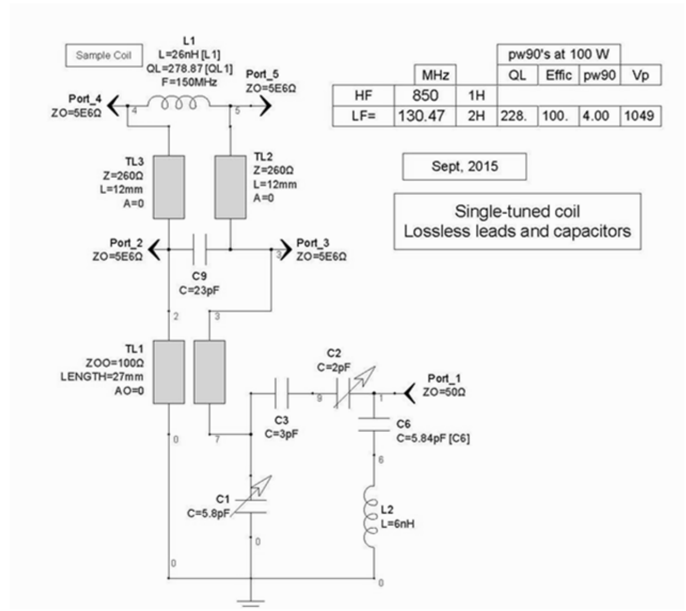

Figure 3.3. The same circuit of Figure 3.1 with lossless tuning components.

The value of c_v in Figure 3.2 was 440. The value c_0 found in Figure 3.4 was 301.

The value of c_0 for an ideal coil scales inversely with f ^0.75 (from eqs. 2.2 and 3.7). So a single calculation of c_0 is sufficient for determination of efficiency at any frequency from any port.

$$c_f = c_0 / f\,\text{^}0.75 \qquad (3.3)$$

The only change is which S parameters are used in the respective appropriate "V" calculations. For example, if the LF is at Port_1, the MF at port_2, the HF at port_3, and the hi-Z coil-monitors are at Port_4 and Port_5, the relevant calculations are:

$$V_L = c_{f1} * \text{mag}(S[1,4]-S[1,5]) \qquad (3.4)$$

$$V_M = c_{f2} * \text{mag}(S[2,4]-S[2,5]) \qquad (3.5)$$

$$V_H = c_{f3} * \text{mag}(S[3,4]-S[3,5]) \qquad (3.6)$$

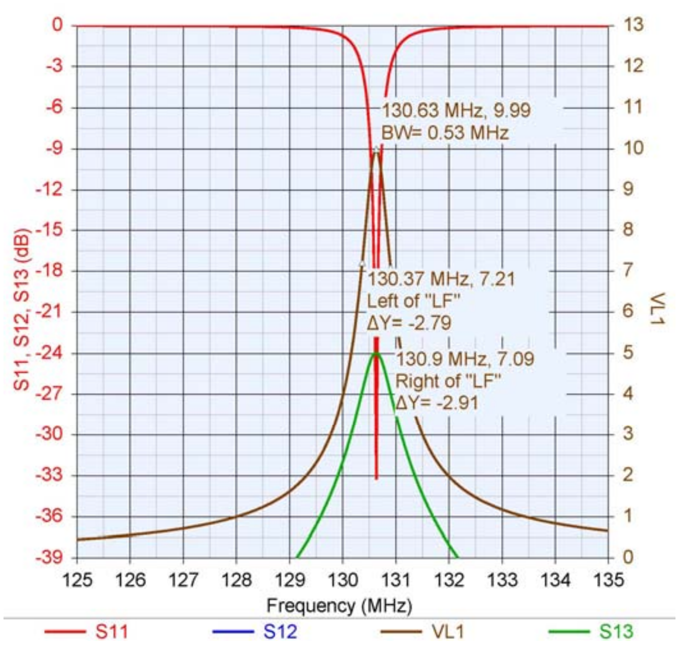

Figure 3.4. Some S parameters and VL1 for the circuit of Figure 3.3.

There are alternatives to the above method that are more intuitive and may work better if one does not need to keep two fixed scales on a graph along with a peak value for the voltage trace that is more than two-thirds of full scale. (In Genesys, it is important that the graphical display of "V" be close to full scale or there can be significant errors in the -3dB bandwidth reported by the BW marker function.)

The voltage across the sample coil for the ideal case is:

$$V_{PI} = \sqrt{2PR_P} \qquad (3.7)$$

where $R_P = Q_{0L}\, X_L$. For the above example, $Q_{0L} = 259$ at 130.5 MHz, so $R_P = 5520\ \Omega$. The result of the above at 100 W agrees with the peak voltage posted in Figure 3.3. The percent efficiency is then simply given by

$$\eta_E = 100 * (V_{PR} / V_{PI})^2 \qquad (3.8)$$

where V_{PR} is the sample coil voltage for the real circuit.

As noted, the “V’s” calculated by the above equations assume a source which delivers 10 mW to a matched 50Ω load, or 1 V peak, which is standard. Equation 2.1 was presented as calculating B_1 for the matched circuit from a source capable of delivering power *P* to a matched load. However, that equation is still valid for the unmatched case using P_i, defined as the power absorbed by the circuit, or $P(S11)^2$. In some cases it can be useful to multiply the V’s calculated by $(S11)^2$ to get a better sense of the circuit efficiency independent of the matching accuracy.

The program attached to the above schematic (Figure 3.3) follows. The only difference for Figure 3.1 was lines 2 and 3, for which LFSF=440 and LFBW=1.13. Note that eq. 3.6 doesn’t appear here. It is in the definition of the variable VL1 in the data file graphed in Figures 3.2 and 3.4.

```
H1 = 850        ' 1H frequency
 LFSF=301      ' scale factor in LF coil V graph for “10”
 LFBW=0.57   ' LF 3db bandwidth from LF graph
 L1 = 26         ' 4 turns, 1.2mm wire
  Coil_ID=4
  Coil_lgh=10
  beta=2.4       ' for sol with l=d, beta=2.1 at MA, or 2.4 for 90-deg;
  QL1a = 300   ' coil Q at 150 MHz
 L2 = 6             ' series 1H trap on Port 1
H2f = 0.1535 * H1   ' H2 freq
LF = H2f         ' LF, MHz
LFw=2*PI*LF*1E6
H1w=2*PI*H1*1E6
C6 =1.05E21 /((L2 +0.3)*H1w^2) '  series 1H trap at port
LFQL=LF/LFBW    ' Note that LFBW must be entered manually
CQS = 1.5       ' Quartz variable stray
QL1 = QL1a *(LF/151)^0.5    ' coil QL at LF
LFgp = LF / H1 * 42.577                ' LF gamma-prime, MHz/T
QL1_LF = QL1 *(LF/150)^0.5        ' QL at LF
 Vp_LF = 316 * 1000/LFSF            ' LF peak coil differential voltage at 100 W
 ' Above assumes the V probe ports are 5E6 ohms
 LFSFC = 1.000E6/(L1*QL1)^0.5    ' fits ideal circuit data at 150 MHz
 SFLF0=0.99* LFSFC/LF^0.75        ' ideal scale factor for "10V"
 LFEff=100*(SFLF0/LFSF)^2           ' LF RF efficiency
 Vc = 3.14 * Coil_ID^2 * Coil_lgh/4000   ' coil volume, mL
 B1_LF = 0.0001 *beta *(LFEff *QL1_LF/(LF * Vc))^0.5    ' mean B1, T, at 100 W
 LFgB1 = 1000 *LFgp * B1_LF         ' kHz, LF rf field strength
 LFpw90 = 250/LFgB1                      ' pi/2, micro-sec
```

In the above schematics, TL1 is a two-wire coupled-transmission-line model, as would be appropriate for two leads closer to each other than to a ground plane, or for a twisted-pair. TL2 and TL3 are single-line TL models, as is appropriate in most other cases. The next chapter will, among other things, describe how one arrives at reasonable estimates of the characteristics of the transmission lines and other parasitic components.

The Genesys files for the two above examples are available for download at the Doty website:

H2_130MHz_4mm.wsx

H2_130MHz_4mm_ideal.wsx

As noted earlier, the first of the above uses a custom model for the capacitors. Hence, it will not run until that is added to the model library. That will be described in the next chapter.